The Curious Case of the Kaprekar Constant
A Mathematical Essay

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Abstract

Kaprekar's routine is a mathematical process that involves iteratively sorting the digits of a number in ascending and descending order, subtracting the smaller number from the larger one, and repeating the process until a fixed point, known as Kaprekar constant, is reached. This essay explores the properties of Kaprekar's routine, its convergence behavior for different digit lengths, and the discovery of Kaprekar constants such as 6174 for four-digit numbers. A computational approach using Python is presented to analyze systematically the routine for 1 to 9-digit numbers, identifying constants and cycles.

Keywords: Kaprekar constant, Kaprekar routine, trivial Kaprekar constant, cycle

1 Introduction

1.1 Definition of Kaprekar Constant

Kaprekar constant, 6174, exhibits a unique mathematical property. If you choose almost any four-digit number, rearrange its digits in descending and ascending order to form two new numbers, subtract the smaller from the larger, and repeat the process, you will eventually reach 6174 within at most seven iterations.[1] Once 6174 is reached, the process becomes self-sustaining:

$$7641 - 1467 = 6174.$$

This iterative process is known as Kaprekar's routine. However, there is an exception: if a number consists of identical digits, the process terminates at zero. For example,

$$1111 - 1111 = 0.$$

Some numbers, such as 1000, initially appear to reach zero, but when leading zeros are considered, the process continues. For instance,

$$1000 - 0001 = 999,$$

followed by

$$999 - 999 = 0.$$

However, treating 999 as 0999 allows the routine to proceed:

$$9990 - 0999 = 8991,$$

which ultimately converges to 6174.

If we take every number from 0 to 9999, 9990 of these numbers will eventually converge to 6174 (if we include leading zeroes). The distribution of the number of iterations needed to reach the Kaprekar constant is shown as a bar graph in Figure 1 below. Note that it can take up to seven iterations for some four-digit numbers.

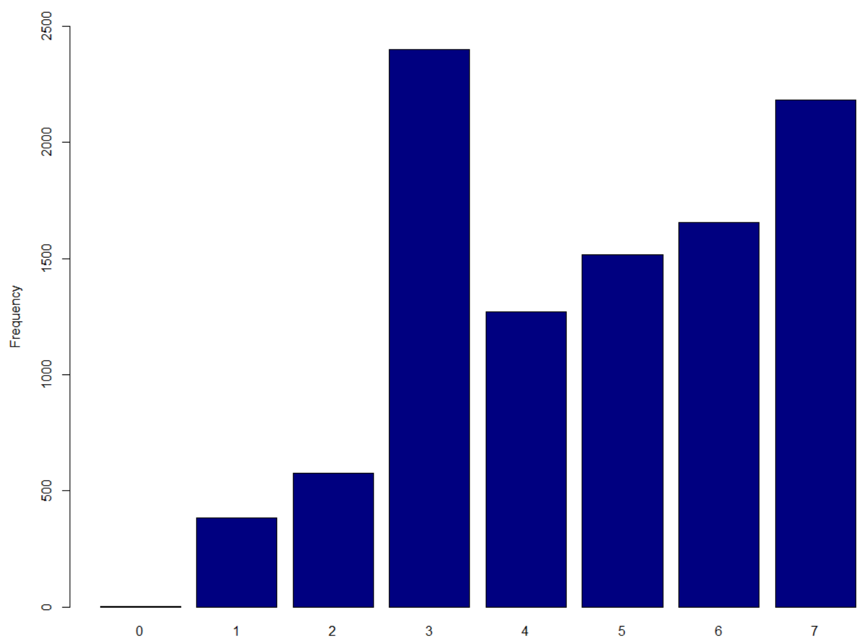

Figure 1: Bar graph for the number of iterations until 6174 appears

[2]

1.2 Historical Note: Dattatreya Ramchandra Kaprekar

Dattatreya Ramchandra Kaprekar was born on January 17, 1905, in Dahanu, Maharashtra, India. He developed a passion for mathematics at a young age and later studied at Fergusson University in Pune, a prestigious institution then and now. After earning a Bachelor's Degree in Science in 1929, he became a school teacher in Devlali, where he worked for the next 33 years.

Despite having no formal postgraduate research, D.R. Kaprekar devoted his free time to number theory. In 1946, he discovered that taking a four-digit number with at least two distinct digits, rearranging its digits to form the largest and smallest possible values, and subtracting the smaller number from the larger would eventually lead to 6174—now known as the Kaprekar constant. He published this discovery in 1955, along with other contributions to number theory.[3]

After retiring in 1962, D. R. Kaprekar continued teaching mathematics privately, as his pension was insufficient. In 1975, Martin Gardner popularized Kaprekar constant, bringing it to international fame through his ”Mathematical Games” column in Scientific American.[4] D. R. Kaprekar passed away on June 17, 1986.

2 Kaprekar Constants Beyond Four Digits

What happens if we test Kaprekar's routine on numbers with different digit lengths? The answer has been well-researched in [5]. Here are my results after creating a Python program to test this.

• **One-digit numbers:** Subtracting the number from itself results in 0, so the Kaprekar constant in this case is 0.

Note that for any number consisting of identical digits, regardless of its length, the Kaprekar routine leads to zero, which, as a Kaprekar constant, is referred to as a trivial Kaprekar constant.

• **Two-digit numbers:** Instead of a single Kaprekar constant, there are five cyclic Kaprekar constants. If you perform Kaprekar's routine on one of these numbers, it will eventually cycle back to the original number. For example, $81 \to 63 \to 27 \to 45 \to 09 \to 81$. There is no single fixed Kaprekar constants for two-digit numbers, and there are four cyclic Kaprekar constants: 81, 63, 45, and 27.

• **Three-digit numbers:** Most numbers lead to 495, the Kaprekar constant for three-digit numbers. There is one fixed Kaprekar constant and no cyclic Kaprekar constants.

The number of iterations in the Kaprekar routine for three-digit numbers is presented as a bar graph in Figure 2 below.

• **Four-digit numbers:** The process generally converges to 6174, the famous Kaprekar constant for four-digit numbers. There is one fixed Kaprekar constant and no cyclic Kaprekar constants.

• **Five-digit numbers:** There are 10 cyclic Kaprekar constants and no fixed Kaprekar constants.

• **Six-digit numbers:** There are 7 cyclic Kaprekar constants and 2 fixed Kaprekar constants.

• **Seven-digit numbers:** There are 8 cyclic Kaprekar constants and no fixed Kaprekar constants.

• **Eight-digit numbers:**; There are 10 cyclic Kaprekar constants and 2 fixed Kaprekar constant.

• **Nine-digit numbers:** There are 14 cyclic Kaprekar constants and 2 fixed Kaprekar constant.

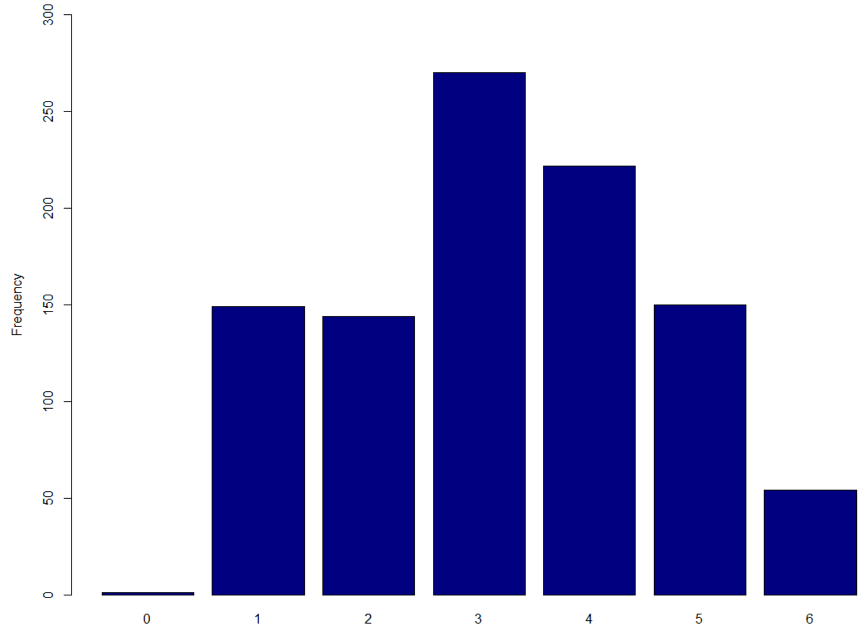

Figure 2: Bar graph for the number of iterations until 495 appears

- **Ten-digit numbers:** There are 16 cyclic Kaprekar constants and 1 fixed Kaprekar constant.

3 Lesser-Known Properties of Kaprekar Constants

The Kaprekar constants possess intriguing properties that are not widely known.

3.1 Divisibility by 9

Every Kaprekar constant, regardless of the number of digits, is always divisible by 9.[6] In fact, this property holds for any number obtained by applying Kaprekar's routine. We prove it for 4-digit numbers. Let $ABCD$ be a four-digit number with digits A, B, C, and D such that $A > B > C > D$. Applying Kaprekar's routine, we obtain:

$$ABCD - DCBA = 1000 \cdot A + 100 \cdot B + 10 \cdot C + D - 1000 \cdot D - 100 \cdot C - 10 \cdot B - A$$

$$= 999 \cdot A + 90 \cdot B - 90 \cdot C - 999 \cdot D = 9(111 \cdot A + 10 \cdot B - 10 \cdot C - 111 \cdot D),$$

which is divisible by 9.

3.2 Middle Digit

For every odd-digit Kaprekar constant, the middle digit is always 9. This follows from the borrowing process inherent in Kaprekar's routine. When the difference between the largest and smallest possible permutations of the digits is computed, borrowing cascades to the middle digit. The middle-digit calculation results in: $10 - 1 + (digit) - (digit) = 9$.

For illustration, we present the proof for Kaprekar's routine applied to a five-digit number. Consider a five-digit number $ABCDE$ with digits in descending order, $A > B > C > D > E$. Applying Kaprekar's routine results in the difference:

$$ABCDE - EDCBA.$$

Examining the rightmost digits, we see that

• The fifth digit (rightmost place) is $E - A$, which requires borrowing from the fourth digit since $E < A$, making it $10 + E - A$.

• The fourth digit becomes $D - 1 - B$, but again, since $D < B$, borrowing is needed from the third (middle) digit, making it $10 + D - 1 - B = 9 + D - B$.

• The middle digit becomes $C - 1 - C$. Since this is negative, borrowing from the second digit is required, leading to: $10 + C - 1 - C = 9$.

Thus, for any odd-digit number processed through Kaprekar's routine, the middle digit of the result is always 9. Note that this pattern holds for all odd-digit Kaprekar's constants listed in Section 7.

3.3 Sum of Digits

If we sum the digits of numbers after applying Kaprekar's routine, we observe similar patterns. We stopped at five digits because there are multiple cases for the digits beyond five digits.

• **One-digit numbers:** Whenever you apply Kaprekar's routine to a 1 digit number, this will always equal 0 because any number subtracted by any number equals 0.

• **Two-digit numbers:** Whenever you apply Kaprekar's routine to a 2 digit number with digits AB (and $9 >= A > B >= 0$), the digits of the new number, CD, will equal $AB - BA$. $D = 10 + B - A$, and $C = A - B - 1$. If we $C + D$, we will get 9. Therefore, whenever you apply Kaprekar's routine to a 2 digit number, the digits of the new number will always add up to 9.

• **Three-digit numbers:** Whenever you apply Kaprekar's routine to a 3 digit number with digits ABC (and $9 >= A >= B >= C >= 0$, and A is not equal to C), the digits of the new number, DEF, will equal $ABC - CBA$. $F = 10 + C - A, E = 10 + B - B - 1 = 9$, and $D = A - C - 1$. Therefore, whenever you apply Kaprekar's routine to a 3 digit number, the middle digit will always be 9 (as proven in section 3.2), and the other two digits will always add up to 9 ($D + F = A - C - 1 + 10 + C - A = 9$).

• **Four-digit numbers:** Whenever you apply Kaprekar's routine to a 4 digit number with digits $ABCD$ (and $9 >= A >= B >= C >= D >= 0$, and A is not equal to D), the digits of the new number, $EFGH$, will equal $ABCD - DCBA$. There are two cases. If $B = C$, then $H = 10 + D - A, G = 10 + C - B - 1 = 9, F = 10 + B - C - 1 = 9$, and $E = A - D - 1$. Therefore, if you apply Kaprekar's routine to a number $ABCD$ and $B = C$, then the second and third digits will always equal 9, and the first and fourth digits will always add up to 9 ($E+H = A-D-1+10+D-A = 9$).

However, if B is not equal to C, then $H = 10+D-A, G = 10+C-B-1 = 9+C-B, F = B-C-1$, and $E = A - D$. Therefore, whenever you apply Kaprekar's routine to a number $ABCD$ and B is not equal to C, then the second and third digits will always add up to 8 ($F+G = B-C-1+9+C-B = 8$), and the first and fourth digits will always add up to 10 ($E + H = A - D + 10 + D - A = 10$).

• **Five-digit numbers:** Whenever you apply Kaprekar's routine to a 5 digit number with digits $ABCDE$ (and $9 >= A >= B >= C >= D >= E >= 0$, and A is not equal to E), the digits of the new number, $FGHIJ$, will equal $ABCDE - EDCBA$. There are two cases. If $B = D$, then $J = 10 + E - A, I = 10 + D - B - 1 = 9, H = 10 + C - C - 1 = 9, G = 10 + B - D - 1 = 9$, and $F = A - E - 1$. Therefore, if you apply Kaprekar's routine to a number $ABCDE$ and $B = D$, then the middle digit will always equal 9 (as proven in Section 3.1), the second and fourth digits will always equal 9, and the first and fifth digits will always add up to 9 ($F+J = A-E-1+10+E-A = 9$).

However, is B is not equal to D, then $J = 10 + E - A, I = 10 + D - B - 1 = 9 + D - B, H = 10 + C - C - 1 = 9, G = B - D - 1$, and $F = A - E$. Therefore, if you apply Kaprekar's routine to a number $ABCDE$ and B is not equal to D, then the middle digit will always equal 9 (as proven in Section 3.1), the second and fourth digits will always add up to 8 ($G+I = B-D-1+9+D-B = 8$), and the first and fifth will always add up to 10 ($F + J = A - E + 10 + E - A = 10$).

Further investigation revealed that for six and seven digits, there are four cases; for eight and nine digits, there are eight cases; for ten and eleven digits, there are sixteen cases; and so on. We hypothesize that the number of cases increases exponentially as the number of digits grows.

3.4 One-Time Iteration on Numbers

If we simply perform Kaprekar's routine only ONE time on numbers, there are some interesting results. For every digit number, it is found that it will lead to a specific group of numbers. After applying it multiple times will it narrow down to only the Kaprekar constants.

• **One-digit numbers:** Every time you apply Kaprekar's routine to a one-digit number, it will always equal zero, so therefore there is only one number in the specific group: 0.

• **Two-digit numbers:** Every time you apply Kaprekar's routine to a two-digit number with digits AB, the result becomes $AB - BA = 10A + B - 10B - A = 9A - 9B = 9(A - B)$, and the maximum $A - B$ can equal is 9. Because of this, the result must be divisible by 9. There are ten numbers in this specific group: 0, 09, 18, 27, 36, 45, 54, 63, 72, 81.

• **Three-digit numbers:** Every time you apply Kaprekar's routine to a three-digit number with digits ABC, the result becomes $ABC - CBA = 100A + 10B + C - 100C - 10B - A = 99A - 99C = 99(A - C)$, and the maximum $A - B$ can equal is 9. Because of this, the result must be divisible by 9 and cannot be greater than 99 x 9 = 891. There are ten numbers in this specific group: 0, 99, 198, 297, 396, 495, 594, 693, 792, and 891.

• **Four and five-digit numbers:** For both four- and five-digit numbers, the size of numbers in the specific group is equal to 55.

• **Six and seven-digit numbers:** For both six- and seven-digit numbers, the size of numbers in the specific group is equal to 220.

• **Eight and nine-digit numbers:** For both eight- and nine-digit numbers, the size of numbers in the specific group is equal to 715.

• **Ten-digit numbers:** For ten-digit numbers, the size of numbers in the specific group is equal to 2002.

A pattern can be found. For $2n$ and $2n + 1$ digit numbers (where n >= 1, the size of numbers in the specific group is the same. Table 1 below displays the selected groups for four-digit numbers.

Table 1. Specific Groups of Four-Digit Numbers

0	0999	1089	1998	2088
2178	2997	3087	3177	3267
3996	4086	4176	4266	4356
4995	5085	5175	5265	5355
5445	5994	6084	6174	6264
6354	6444	6534	6993	7083
7173	7263	7353	7443	7533
7623	7992	8082	8172	8262
8352	8442	8532	8622	8712
8991	9081	9171	9261	9351
9441	9531	9621	9711	9801

It can be easy to prove why it only leads to a specific group of numbers. Whenever Kaprekar's routine is applied to a number, it must be divisible by 9 (as proven in Section 3.1), and if the number of digits is odd, then the middle digit must be 9 (as proven in Section 3.2).

4 Proof of Existence and Uniqueness of Kaprekar Constant 6174

4.1 Proof of Existence

Among the 9000 four-digit numbers, 8991 numbers eventually reach 6174 when Kaprekar's routine is applied, in at most 7 iterations. This excludes the 9 numbers for which all digits are equal: 1111 through 9999. The highest possible difference obtained using Kaprekar's routine is $9990 - 0999 = 8991$ and the smallest possible is $1000 - 0001 = 999$ (excluding 0). Since the sequence of transformations is finite and the number space is bounded, the iterations must eventually enter a cycle. We now establish that 6174 is the only number in such a cycle.

4.2 Proof of Uniqueness

Let $ABCD$ be a four-digit number, where A, B, C, D are its digits, ordered such that $9 \geq A \geq B \geq C \geq D \geq 0$, and at least two of them are distinct (to exclude cases where Kaprekar's routine yields zero). Applying Kaprekar's routine, we get $ABCD - DCBA$.

- The fourth digit place will be $10 + D - A$ (because $A > D$), and A cannot equal D because if $A = D$, then $A = B = C = D$, which is not possible.

- The third digit place will be $C - 1 - B + 10 = 9 + C - B$ because we had to borrow for the fourth digit place, and $B > C$.

- The second digit place will be $B - C - 1$ because we had to borrow for the third digit place (if $B = C$, then the digit will be $10 + B - C - 1 = 9 + B - C$).

- The first digit place will be $A - D$ (if $B = C$, then the first digit place will be $A - D - 1$).

The number $ABCD$ will cycle under Kaprekar's routine if the result can be written using the same digits A, B, C, and D. Thus, the resulting digits $10 - D - A$, $9 + C - B$, $B - C - 1$, and $A - D$ (or $A - D - 1$ if $B = C$) must be a permutation of A, B, C, and D, under the conditions that $9 >= A >= B >= C >= D >= 0$, and A, B, C, and D are not all equal.

4.2.1 Assuming B = C

Let's assume that B = C. The digits of the new number will be E, F, G, and H. Using the formulas:

$E = A - D - 1$
$F = 10 - 1 + B - C = 9$ (because B = C)
$G = 10 - 1 + C - B = 9$ (because B = C)
$H = 10 + D - A$
We know that $F = G = 9$, and that $E + H = A - D - 1 + 10 + D - A = 9$. Because 9 is the biggest digit, that means we can substitute A and B as 9, because they are the highest digits. Because $B = C$, that

means C also equals 9. Because $E+H=9$, and we know that either E or $H=9$, that means the other one = 9 - 9 = 0. So, $ABCD=9990$. However, when we try to confirm this, we get $9990-999=8991$. Because 8 and 1 are not in (A, B, C, D), that means this is not a Kaprekar constant. So, B must not equal C.

4.2.2 Assuming B is not equal to C

Now, let's assume that B is not equal to C. So, this means $9>=A>=B>C>=D$. Now we can substitute.
$E=A-D$
$F=B-C-1$
$G=10-1+C-B=9+C-B$
$H=10+D-A$

We know that $E+H=10$ and $F+G=8$.

If we assume $(E,H)=(1,9)$, then that means that $A=9$ (because A is the highest possible digit). So, if $H=10+D-A$, then $9=10+D-9=1+D$, so $D=8$. However, this would make $B=C$ (either $B=C=9$, or $B=C=8$). So, this is not possible.

If we assume $(E,H)=(9,1)$, then that means $A=9$. So, if $H=10+D-A$, then $1=10+D-9$, so $D=0$. This would mean that 1, which is H, must be in (B,C). We also know that 0, which is D, must be either F or G, so that means the other one must equal 8 - 0 = 8. So, $ABCD=9810$. If we try to find F and G, we get $F=8-1-1=6$, and $G=9+1-8=2$. However, (6, 8) is not equal to (0, 8) or (8, 0). So, (E,H) must not equal (9, 1).
If we assume (E,H) = (2, 8) or (8, 2), then (F,G) equals either (0, 8), (1, 7), (2, 6), (3, 5), (4, 4), or a permutated version of these.

If we assume (F,G) = (0, 8) or (8, 0), then that means $ABCD=8820$. If we try to find F and G, we get $F=8-2-1=5$, and $G=9+2-8=3$. However, (5, 3) is not equal to (0, 8) or (8, 0). So, this is not possible.

If we assume (F,G) = (1, 7) or (7, 1), then that means $ABCD=8721$. If we try to find F and G, we get $F=7-2-1=4$, and $G=9+2-7=4$. However, (4, 4) is not equal to (1, 7) or (7, 1). So, this is not possible.

If we assume (F,G) = (2, 6) or (6, 2), then that means $ABCD=8622$. If we try to find F and G, we get $F=6-2-1=3$, and $G=9+2-6=5$. However, (3, 5) is not equal to (2, 6) or (6, 2). So, this is not possible.

If we assume (F,G) = (3, 5) or (5, 3), then that means $ABCD=8532$. If we try to find F and G, we get $F=5-3-1=1$, and $G=9+3-5=7$. However, (1, 7) is not equal to (3, 5) or (5, 3). So, this is not possible.

If we assume $(F, G) = (4,\ 4)$, then that means $ABCD = 8442$. However, this makes $B = C$, which is not allowed. So, this is not possible. Because we went through all of the possibilities, this means (E, H) can not be equal to (2, 8) or (8, 2).

If we assume $(E, H) = (3,\ 7)$ or $(7,\ 3)$, then (F, G) either equals (0, 8), (1, 7), (2, 6), (3, 5), (4, 4), or a permutated version of these.

If we assume $(F, G) = (0,\ 8)$ or $(8,\ 0)$, then that means $ABCD = 8730$. If we try to find F and G, we get $F = 7 - 3 - 1 = 3$, and $G = 9 + 3 - 7 = 5$. However, (3, 5) is not equal to (0, 8) or (8, 0). So, this is not possible.
If we assume $(F, G) = (1,\ 7)$ or $(7,\ 1)$, then that means $ABCD = 7731$. If we try to find F and G, we get F = 7 - 3 - 1 = 3, and $G = 9 + 3 - 7 = 5$. However, (3, 5) is not equal to (1, 7) or (7, 1). So, this is not possible.

If we assume $(F, G) = (2,\ 6)$ or $(6,\ 2)$, then that means $ABCD = 7632$. If we try to find F and G, we get $F = 6 - 3 - 1 = 2$, and $G = 9 + 3 - 6 = 6$. This confirms $F = 2$ and $G = 6$. If we try to find E and H, we get $E = 7 - 2 = 5$, and $H = 10 + 2 - 7 = 5$. However, (5, 5) is not equal to (3, 7) or (7, 3). So, this is not possible.

If we assume $(F, G) = (3,\ 5)$ or $(5,\ 3)$, then that means $ABCD = 7533$. If we try to find F and G, we get $F = 5 - 3 - 1 = 1$, and $G = 9 + 3 - 5 = 7$. However, (1, 7) is not equal to (3, 5) or (5, 3). So, this is not possible.

If we assume $(F, G) = (4,\ 4)$, then that means $ABCD = 7443$. However, this makes $B = C$, which is not allowed. So, this is not possible. Because we went through all of the possibilities, this means (E, H) can not be equal to (3, 7) or (7, 3).

If we assume $(E, H) = (4,\ 6)$ or $(6,\ 4)$, then (F, G) either equals (0, 8), (1, 7), (2, 6), (3, 5), (4, 4), or a permutated version of these.

If we assume $(F, G) = (0,\ 8)$ or $(8,\ 0)$, then that means $ABCD = 8640$. If we try to find F and G, we get $F = 6 - 4 - 1 = 1$, and $G = 9 + 4 - 6 = 7$. However, (1, 7) is not equal to (0, 8) or (8, 0). So, this is not possible.
If we assume $(F, G) = (1,\ 7)$ or $(7,\ 1)$, then that means $ABCD = 7641$. If we try to find F and G, we get $F = 6 - 4 - 1 = 1$, and $G = 9 + 4 - 6 = 7$. This confirms $F = 1$ and $G = 7$ (this makes it (1, 7), not (7, 1)). If we try to find E and H, we get $E = 7 - 1 = 6$, and $H = 10 + 1 - 7 - 4$. This confirms that $E = 6$ and $H = 4$ (this makes it (6, 4), not (4, 6). Therefore, the number 6174 (which is the Kaprekar constant) meets all of the requirements.

If we assume $(F, G) = (2,\ 6)$ or $(6,\ 2)$, then that means $ABCD = 6642$. If we try to find F and G, we get $F = 6 - 4 - 1 = 1$, and $G = 9 + 4 - 6 = 7$. However, (1, 7) is not equal to (2, 6) or (6, 2). So, this is not possible.

If we assume $(F, G) = (3,\ 5)$ or $(5,\ 3)$, then that means $ABCD = 6543$. If we try to find F and G, we get $F = 5 - 4 - 1 = 0$, and $G = 9 + 4 - 5 = 8$. However, (0, 8) is not equal to (3, 5) or (5, 3). So, this is not possible.

If we assume $(F, G) = (4,\ 4)$, then that means $ABCD = 6444$. However, this makes $B = C$, which is not allowed. So, this is not possible. Because we went through all the possibilities, this means that only one set of conditions, where $E = 6, F = 1, G = 7$, and $H = 4$, or 6174, will work.

If we assume $(E, H) = (5,\ 5)$, then this means that $A = B = 5$, or $C = D = 5$, because B must not equal C. If $C = D = 5$, that means (A, B) is a permutation of (F, G), and $A >= B > 5$. However, this means that $A + B = F + G > 10$, and it is not possible for it to equal 8. So, this means that $A = B = 5$. Because $E = A - D$, that means $5 = 5 - D$, so $D = 0$. Because D is in (F, G), this means that $F + G = 8$, and because either F or G is 0, that means the other one is 8. So, this would make ABCD = 8550. However, this makes $B = C$, which is not possible. So, (E, H) is not equal to (5, 5).

Out of all the possibilities we went through, the only number that works is 6174. So, this is proven.

5 Proof of Existence and Uniqueness of 495

Here, we will use the same method we did for Section 4 to prove that 495 is the only 3-digit Kaprekar constant.

We can use the same proof as in Section 4.1 to prove why all numbers must eventually repeat. The highest possible difference after using Kaprekar's routine to three digit numbers is 891, by using 990 - 099 = 891, and the lowest possible difference we get is 99, by using 998 - 899 = 99 (excluding 0). Because the number space is bounded, this means that after doing Kaprekar's routine a certain amount of times, it will eventually converge at a number, or will make a certain number repeat.

Let's say that the number ABC is a three digit number, where $9 >= A >= B >= C$, and A is not equal to C. When we apply Kaprekar's routine to ABC, we get $ABC - CBA$. Let's call this difference DEF, where D, E, and F are its digits. $F = 10 + C - A, E = 10 + B - B - 1 = 9$ (as proven in section 3.2), and $D = A - C - 1$. We also know that A must equal 9 because $E = 9$, E is either A, B, or C, and 9 is the biggest one digit number.

If we assume that $(D, F) = (0, 9)$ or $(9, 0)$, then that means $A = B = 9$, and $C = 0$. However, $D = A - C - 1 = 9 - 0 - 1 = 8$, which goes against our assumption. Therefore, (D, F) is not equal to (0, 9) or (9, 0).

If we assume that $(D, F) = (1, 8)$ or $(8, 1)$, then that means $B = 8$, and $C = 1$. However, $D = A - C - 1 = 9 - 1 - 1 = 7$, which goes against our assumption. Therefore, (D, F) is not equal to (1, 8) or (8, 1).

If we assume that $(D, F) = (2, 7)$ or $(7, 2)$, then that means $B = 7$, and $C = 2$. However, $D = A - C - 1 = 9 - 2 - 1 = 6$, which goes against our assumption. Therefore, (D, F) is not equal to (2, 7) or (7, 2).

If we assume that $(D, F) = (3, 6)$ or $(6, 3)$, then that means $B = 6$, and $C = 3$. However, $D = A - C - 1 = 9 - 3 - 1 = 5$, which goes against our assumption. Therefore, (D, F) is not equal to (3, 6) or (6, 3).

If we assume that $(D, F) = (4, 5)$, then that means $B = 5$, and $C = 4$. So, $D = A - C - 1 = 9 - 4 - 1 = 4$, and $F = 10 + C - A = 10 + 4 - 9 = 5$, which confirms this set. That means the number that repeats when Kaprekar's routine is applied to it has $D = 4$, $E = 9$, and $F = 5$, or 495.

If we assume that $(D, F) = (5, 4)$, then that means $B = 5$, and $C = 4$. However, $D = A - C - 1 = 9 - 4 - 1 = 4$, which goes against our assumption. Therefore, (D, F) is not equal to (5, 4).

This proves that the only three-digit number that will repeat itself when Kaprekar's routine is applied to it is 495.

6 Applications of Kaprekar Constant

The Kaprekar constant has many lesser-known but interesting applications used for different fields of mathematics.[7] These include the fields of cryptography, convergence analysis, recursive functions, and possibly even more.

Kaprekar's routine can be used for cryptography and prime numbers. For cryptography, it is very useful because it can be used to generate pseudo-random numbers, which can be used for encryption and decryption. Encryption and decryption can also be used for data security. This is necessary because it prevents hackers or other people from spying on private messages. This is very important because we do this regularly.

Kaprekar's routine can also be used for convergence analysis and recursive functions. Because Kaprekar's routine is a function that eventually converges to one specific value, 6174, or the Kaprekar constant (only for 4-digit numbers), Kaprekar's routine can be used to analyze the convergence of other convergent functions. This is the same for recursive functions because Kaprekar's routine is a recursive function.

Because the Kaprekar constant has not been researched enough, there is not a lot of data surrounding its applications. More research should be done around its applications because they can be very useful for other fields of math and for our daily lives.

7 Conclusion

To conclude, almost every four-digit number will eventually go to 6174 when the Kaprekar constant is applied. The only numbers that do not converge to 6174 are numbers with the same digit, like 1111 or 2222. These numbers will always lead to 0. Zero is the only Kaprekar constant where any digit number has 10 numbers that converge to 0. There are also other Kaprekar constants for numbers other than four-digit numbers, and there is more than one Kaprekar constant for five- or more-digit numbers. If leading zeros are considered, this will change the number of digits in certain numbers, and when Kaprekar's routine is applied, the convergent will change depending on the number of leading zeros.

Table 2 below contains the list of all Kaprekar constants for numbers of one through nine digits. The Python code used to produce these results can be found here:

https://github.com/MathStudent11626?tab=repositories.

Table 2. List of All Kaprekar Constants Found for 1- to 9-digit Numbers

0	09	27	45	63
81	495	6174	53955	59994
61974	62964	63954	71973	74943
75933	82962	83952	420876	549945
631764	642654	750843	840852	851742
860832	862632	7509843	7519743	7619733
8429652	8439552	8649432	8719722	9529641
43208766	63317664	64308654	64326654	75308643
83208762	84308652	85317642	86308632	86326632
86526432	97508421	554999445	753098643	762098733
763197633	844296552	863098632	864197532	865296432
865395432	873197622	874197522	883098612	954197541
964395531	965296431	976494321	4332087666	6431088654
6433086654	6433266654	6543086544	7533086643	8321088762
8332087662	8433086652	8533176642	8633266632	8653266432
8655264432	8732087622	8765264322	9751088421	9753086421
9755084421	9775084221			

8 Other Curious Numbers

There are several types of numbers that have special properties. **Perfect numbers** are numbers that are equal to the sum of their proper divisors (excluding the number itself). For example, 6 is a perfect number because its divisors are 1, 2, 3, and 6, and 1 + 2 + 3 = 6 (excluding 6). Another perfect number is 28, because its divisors are 1, 2, 4, 7, 14, and 28, and 1 + 2 + 4 + 7 + 14 = 28 (excluding 28). All known perfect numbers are even, and many people are trying to prove whether an odd perfect number exists. This is a well-known unsolved problem in mathematics. [8]

Vampire numbers are numbers with an even number of digits that can be factored into two numbers, whose digits are a rearrangement of the original number. For example, 1260 is a vampire number because $21 \times 60 = 1260$, and 1395 is a vampire number because $15 \times 93 = 1395$. [9]

Kaprekar numbers are numbers that, when squared, can be split into two parts that add up to the original number. For example, 45 is a Kaprekar number because 45 squared = 2025, and 20 + 25 =

45. Similarly, 99 is a Kaprekar number because 99 squared = 9801, and 98 + 01 = 99. [10]

Mersenne primes are prime numbers that can be written in the form $2^n - 1$. For example, 3 is a Mersenne prime because 3 can be written as $2^2 - 1$. Another Mersenne prime is 31 because 31 can be written as $2^5 - 1$. Mersenne primes are very rare, and there are only 53 known Mersenne primes. [10]

Armstrong numbers are numbers such that it is equal to the sum of its digits each raised to the power of the number of digits. For example, 153 is an Armstrong number because 153 can be written as $1^3 + 5^3 + 3^3$. Another Armstrong number is 370 because 370 can be written as $3^3 + 7^3 + 0^3$. [10]

Taxicab numbers are numbers such that the nth taxicab number can be written as the sum of two cubes in n distinct ways. The most famous taxicab number is 1729. 1729 can be written as the sum of $9^3 + 10^3 = 1^3 + 12^3$. This number was discovered by Srinivasi Ramanujan in 1919. [11]

These numbers each exhibit unique properties, such as the Kaprekar constant, and further research into them could yield important insights in the field of mathematics.

Acknowledgments

The author would like to express sincere gratitude to Dr. Olga Korosteleva, Professor of Statistics at California State University, Long Beach, for her valuable guidance in advanced mathematics and her generous support with references and editorial suggestions for this paper.
The author is also deeply thankful to my school teachers and college professors, whose dedication and instruction have laid the foundation for much of my knowledge today.
Finally, the author extends heartfelt appreciation to my parents for their unwavering support throughout the publication process and for providing me with the opportunity to learn from exceptional educators who have enabled me to advance my studies.

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