# Stochastic Modeling of Predator Movements with Lévy Flight Processes

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**Abstract**

This study uses mathematical modeling to compare predator movement in air, sea and grassland habitats with kernel density estimation and Lévy flight models. The results show that mathematical methods can reflect patterns in animal behavior and highlight the impact of the environment on movement.

**Keywords**: predator movement, Lévy flight process, Gaussian kernel

# 1 Introduction

## 1.1 Background

Survival. The critical goal of all animals – no matter where on the planet they live, no matter what environment they thrive in. Many survival needs, such as foraging for food, avoiding predators, and tracking prey all require efficient movement. Predators, in particular, must be able to hunt and search for prey while moving to survive. Unlike animals that look for passive food sources, predators are actively influenced by their prey's movements. As a result, predator movement varies greatly depending on the type of prey they chase and the environment where they hunt.

The central aim of this project is to compare mathematically the movement of three predators that primarily operate in distinct environments – open air, sea, and grassland. In this

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article, we analyze sample trajectories of an albatross, a sea turtle, and a jaguar, comparing them based on the distributions of straight-line movement lengths and turning angles. The general theoretical framework of this article is based on the concept of Lévy flight, a random walk in which the step lengths follow a heavy-tailed probability distribution and the directions of the steps are randomly chosen.

## 1.2 Literature Review

Many past studies, including Reynolds (2014) and Paun et al. (2022), have gathered and analyzed data on predator movements in different habitats. From the open air to the ocean to the grasslands, these animals evolve unique methods of movement over time for efficiency and survival. GPS monitoring is increasingly being used to capture movement data, which is then analyzed using both observational and statistical methods to better understand how predators act in their habitats. This raises the question of whether or not predator movement in different environments can be compared using mathematical methods.

In particular, many previous studies have modeled animal movement using mathematical strategies such as random walks and Levy flights. Viswanathan et al. (1999) suggested that animals such as albatrosses, deer, and bumblebees show Levy flight behavior in their search for food. However, further reanalysis by Edwards et al. (2007) questioned this conclusion, finding that the original approach did not fully demonstrate power-law scaling in the data. Instead, they advocated gamma distributions as a better fit. These studies emphasize the need to use appropriate statistical tools and models when conducting ecological research.

Furthermore, comparing predators from different environments has revealed major adaptations to their surroundings. Marine animals, for example, often show longer movements, possibly due to their open and free habitats (Edwards et al., 2007), while land animals live in a more complex landscape, resulting in shorter steps and many direction changes (Paun et al., 2022). These limits based on habitat have an important effect on not just the animals' behaviors, but also the mathematical frameworks that best model their movement. An example of this is Paun et al. (2022), who developed a hierarchical Gaussian process model that accounts for geographic movement features, demonstrating how environmental factors such as vegetation influence large wildebeest migrations.

While several studies have studied predator movement within specific ecosystems, few have directly compared movement across environments with mathematical methods. This project aims to address that gap by analyzing predator trajectories from three different ecosystems and comparing their fit to different mathematical models with a focus on similarities and differences in behavior.

## 2 Mathematical Basis of Lévy Flight

A **Lévy flight** is a type of random walk in which the step lengths follow a heavy-tailed probability distribution known as a **Lévy stable distribution**. This distribution allows for occasional long jumps, which distinguishes Lévy flights from classical Brownian motion, where step lengths are typically normally distributed. Mathematically, the probability distribution $P(l)$ of a step of length $l$ in a Lévy flight often follows a power law of the form:

$$P(l) \sim l^{-\mu} \text{ for } 1 < \mu \leq 3,$$

where $\mu$ is the **Lévy exponent** that determines the tail heaviness of the distribution. When $\mu \leq 2$, the variance of the step lengths is infinite, allowing for very long jumps to occur with non-negligible probability.

In two or higher dimensions, the steps are taken in isotropic (random and directionally uniform) directions.

The "Lévy flight" was named after the French mathematician Paul Pierre Lévy (1886-1971), who made major contributions to the theory of stable distributions. These distributions are characterized by heavy tails and infinite variance, making higher values more likely than in normal distributions. While Lévy's work was initially theoretical, it received widespread scientific interest in the late twentieth century as academics began to see applications of Lévy flights in other fields such as finance, weather, and biology.

# 3 Implementation of Levy Flight

## 3.1 Description of Dataset

Data on the GPS-based movements of Galápagos albatrosses, Hawksbill sea turtles native to the Chagos Archipelago in the western Indian Ocean, and jaguars from the Brazilian Pantanal wetlands were obtained from the public animal movement database Movebank (www.movebank.org), which compiles animal tracking data by researchers worldwide. Each dataset was consisted of a time series of locations, each of which included timestamp, longitude, latitude, and the tag number for each individual animal. These GPS points were recorded at frequent intervals, often within a few hours, over the course of several years.

For this project, one individual was chosen from each predator species to represent what could be the typical movement of its species in its environment. Specifically, albatross #132, turtle #52252, and jaguar #34770 were chosen. These particular individuals were selected due to the fact that their tracked data had relatively continuous trajectories, allowing for a clearer understanding of the animal's general movement patterns, which then improved the ability to compare across different environments–in this case, open air, sea, and grassland. For reference, an example excerpt from the jaguar movement data is displayed below:

| timestamp | longitude | latitude |
|---|---|---|
| 01:53.0 | -57.5383256 | -16.8681778 |
| 01:19.0 | -57.5382839 | -16.8681939 |
| 01:30.0 | -57.5385608 | -16.8680792 |
| 01:37.0 | -57.5394706 | -16.8667131 |
| 01:48.0 | -57.5324050 | -16.8728083 |
| 01:04.0 | -57.5300731 | -16.8736628 |
| 01:11.0 | -57.5299347 | -16.8738936 |
| 01:54.0 | -57.5299708 | -16.8738958 |
| 00:32.0 | -57.5300650 | -16.8737142 |

## 3.2 Comparative Analysis of Trajectories

Figure 1 below depicts the trajectories of the three animals. For each trajectory, we measured the lengths of straight-line movements ("steps") and plotted histograms showing the distribution of the step lengths (see Figure 2 below).

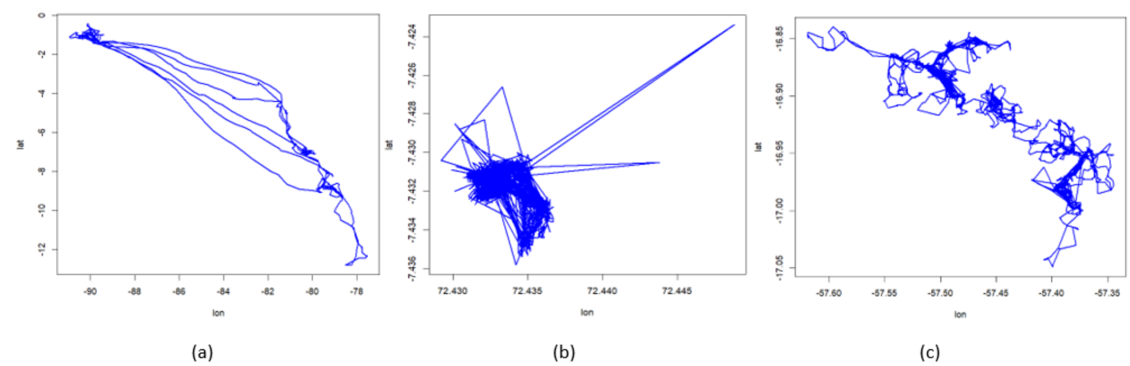


**Figure 1.** Trajectories of (a) albatross, (b) turtle, and (c) jaguar movements.

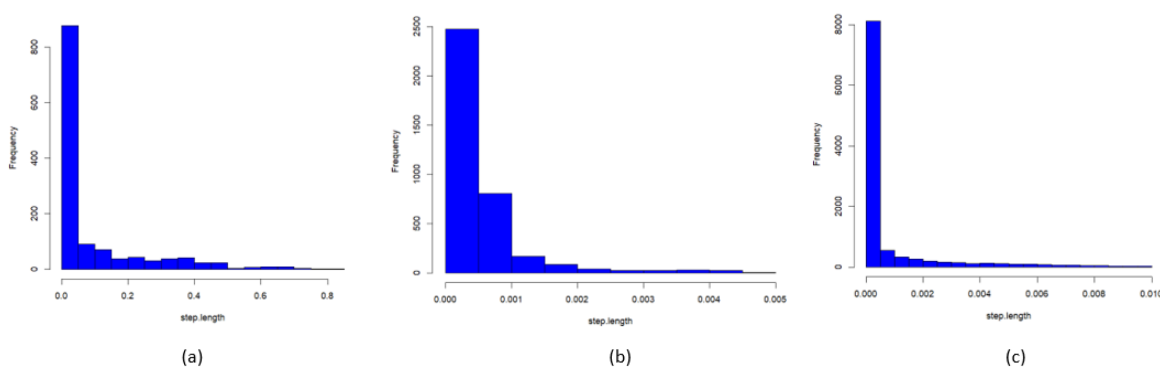


**Figure 2.** Histograms of step length distributions for (a) albatross, (b) turtle, and (c) jaguar.

To verify whether the step length distributions follow the power law prescribed by the Lévy flight, we regressed the frequency on the natural logarithm of mid-bins. The fitted regression slope gives us the negative estimate of the Lévy exponent $\mu$. For the albatross, this estimate is $\hat{\mu} = 2.5924$, whereas for the turtle, it is 413.45, and for the jaguar, it is 250.88.

Further, we measured turning angles, defined clockwise from due south: an angle of $0^\circ$ indicates a leftward turn (around the left shoulder), $360^\circ$ indicates a rightward turn (around the right shoulder), and $180^\circ$ corresponds to continuing straight ahead. Histograms for the distribution of turning angles were generated (see Figure 3 below). To better reveal the underlying distributions of turning angles, a kernel density estimator with a Gaussian kernel, defined by

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right), \quad \text{where} \quad K(u) = \frac{1}{\sqrt{2\pi}}e^{-u^2/2},$$

was used to estimate smooth turning angle distributions across predator species. The resulting densities are illustrated in Figure 4 below.

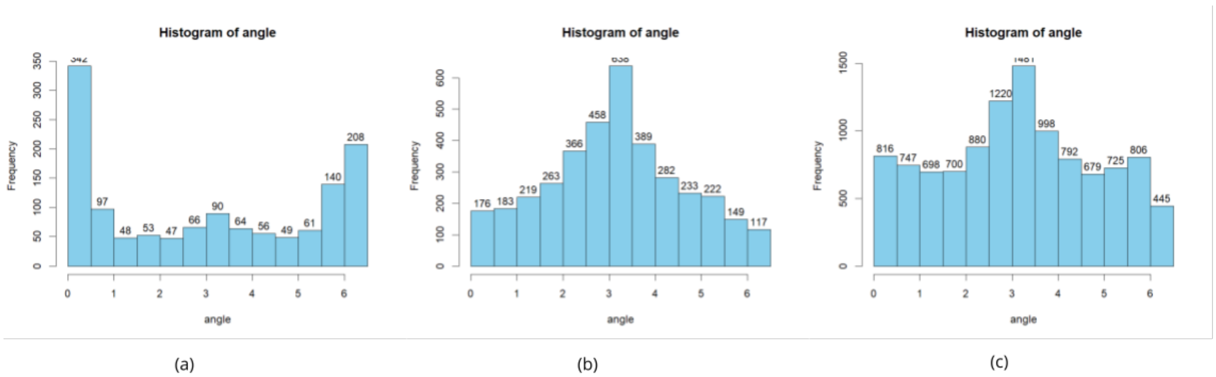


**Figure 3.** Histograms of turning angle distributions for (a) albatross, (b) turtle, and (c) jaguar.

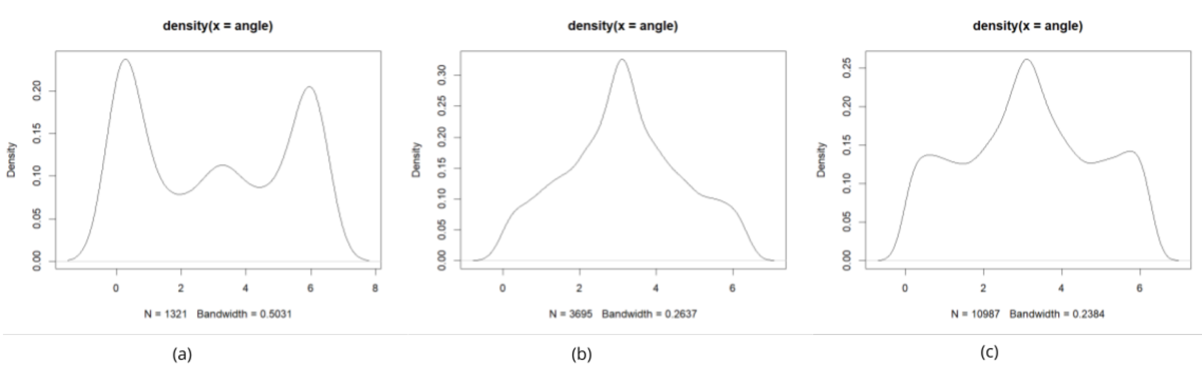


**Figure 4.** Kernel density estimators of turning angle distributions for (a) albatross, (b) turtle, and (c) jaguar.

## 3.3 Results and Discussion

Figure 1 reveals distinct movement patterns for each animal. The albatross clearly travels between two islands in the Galápagos, suggesting purposeful long-distance flight likely tied to nesting. The turtle, on the other hand, appears to swim in a more localized area, perhaps navigating around a coral reef or coastal habitat. In contrast, the jaguar roams widely within its territory, but its path suggests it may be staying relatively close to a water source, consistent with the known behavior of jaguars in the wild.

Studying the histograms of step length distributions shown in Figure 2, we observe that all three distributions decrease with step length. However, only the albatross exhibits a distribution of polynomial order, as indicated by the estimated Lévy exponent $\hat{\mu} = 2.5624$. In contrast, the distributions for the turtle ($\hat{\mu} = 413.45$) and the jaguar ($\hat{\mu} = 250.88$) decay much more rapidly, suggesting an exponential-like drop-off rather than a true power law.

As seen in Figures 3 and 4, the albatross makes frequent sharp turnarounds with no apparent directional preference (left or right). In contrast, the turtle tends to swim in straight paths, making U-turns only rarely. The jaguar shows a more uniform distribution of turning angles, suggesting no strong directional bias in its movement.

So, can we describe the movement of any of the three predators as truly following a Lévy flight? Perhaps not. While each shows some features reminiscent of Lévy-like behavior, their movement patterns deviate from the formal definition. Recall that a true Lévy flight requires turning angles to be isotropic—that is, uniformly distributed over a full circle—and step lengths to follow a power-law decay with an exponent no greater than 3. Below, we summarize the movement characteristics of each predator in more detail.

- **Albatross:** Its step-length distribution follows a power law, consistent with a Lévy flight behavior. However, its turning angles are far from uniform, as it frequently makes abrupt reversals in direction. This violates the isotropy condition.

- **Turtle:** It takes numerous small steps, leading to a steep step-length distribution inconsistent with a power law. Its turning angles show a strong preference for straight paths, with infrequent sharp turns—likely a result of biomechanical and environmental constraints. Its movement does not align with Lévy flight dynamics.

- **Jaguar:** Like the turtle, the jaguar's step-length distribution decays too steeply to be considered Lévy-like. However, among the three species, its turning angles are the closest to being uniformly distributed, spanning the full 360 degrees. Even so, without a supporting power-law step-length distribution, its movement cannot be classified as a true Lévy flight.

### 3.4 Study Limitations and Future Considerations

Several limitations were present in this project, especially regarding the dataset used. One particular limitation was the inability to use more than one individual per species. Due to the need to compare all the histograms one by one, it would have been much more difficult to manage this at the current scale. Thus, the histograms of the animals chosen might not have been fully representative of the entire population. For future work, it would be both interesting and useful to perform this project on a larger scale.

Another limitation this project faced was the nature of the dataset, consisting of coordinate points where animals were tagged. It is possible that animals took other paths between the recorded locations, as the timestamps on the coordinates were often hours apart. The coordinates, therefore, provided only a rough estimate of the animals' general paths over months or years. A higher-resolution tracking system over shorter periods of time would allow for a more precise picture of movement patterns.

Lastly, a major limitation was that the datasets varied both in the length of time each animal was observed for and the number of coordinates that were recorded. These inconsistencies could reflect external influences, such as environmental changes, prey availability, or the animal's well-being, which could then impact movement. These factors could be controlled better with a larger, more uniformly collected dataset across all animals.

## Supplemental Materials

The cleaned datasets and complete R code used in this project can be found on GitHub at this link (https://github.com/xuphoebe/predator-movement-levy-flight.git) for reference.

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