Quantifying NBA Player and Lineup Contributions Using Shapley Values and Convex Optimization

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1 Abstract

It is commonly assumed that in a league like the National Basketball Association (NBA), where statistics are well maintained and broadly reported, the best contributors are clear-cut. The player who gets the most points, rebounds, and assists is typically assumed to be the biggest contributor to team success. While in most cases, these statistics can be indicative of a great player, the four other players are also doing their part to contribute to team success. To obtain a complete picture of player contribution, this study must look beyond traditional statistics to evaluate the offensive and defensive contributions of specific lineups and players using novel frameworks. These frameworks must remain agnostic to the traditional statistics mentioned above, producing data by exclusively observing outcomes. Although this has been attempted previously, many mathematical subtleties have been overlooked. The goal of this project is to provide specific formulas and mathematical rigor to commonly used algorithms for estimating player contribution. This study starts with an exploration of how Shapley Values can be applied to extract individual player contributions from team outcomes. It then presents a novel closed-form solution for ranking teams through convex optimization by drawing on observed results from team matchups. Finally, it explores the contributions of lineups through the same process, equating each lineup to a team and each possession to a game. Beyond basketball, these frameworks provide concrete methods to quantify contribution in any competitive setting

2 Introduction

In the NBA, players such as Shane Battier contribute to team success without relying on traditional statistics like points, rebounds, or assists. This phenomenon is highlighted in Michael Lewis's article "The No-Stat All-Star".[4] A player may directly contribute to scoring by taking a shot, but they may also contribute indirectly by setting a screen or drawing a defender away. Defense is similarly undervalued in conventional metrics, with rebounds, steals, and blocks serving as the only widely recorded indicators of defensive success. This study ranks player and lineup contributions to winning based solely on the outcomes of the plays they take part in, both offensively and defensively.

This subject naturally attracts a mixture of fandom and mathematics. However, the mathematical foundation of existing models are often glossed over. Implementing a reliable ranking system is more challenging than it appears, with many fans struggling to construct robust models or understand the limitations of their algorithms. Even trusted resources such as Basketball Reference lack rigorous algorithms for something as fundamental as NBA team rankings.[2] Moreover, little work has been done to evaluate production at the lineup or player level. This paper demonstrates how techniques like Shapley values and convex optimization can be used to assign credit accurately to individual players or entire lineups, respectively, and rank their contributions.

This study also examines the limitations of individual player models, as basketball is inherently a five-on-five sport. Applying convex optimization to lineups rather than individual players can produce more precise and actionable data.

Finally, this paper addresses the challenges of implementing these models, including data availability and computational complexity, and discusses how their results can be applied.

3 Research

3.1 Methods: What techniques are being used?

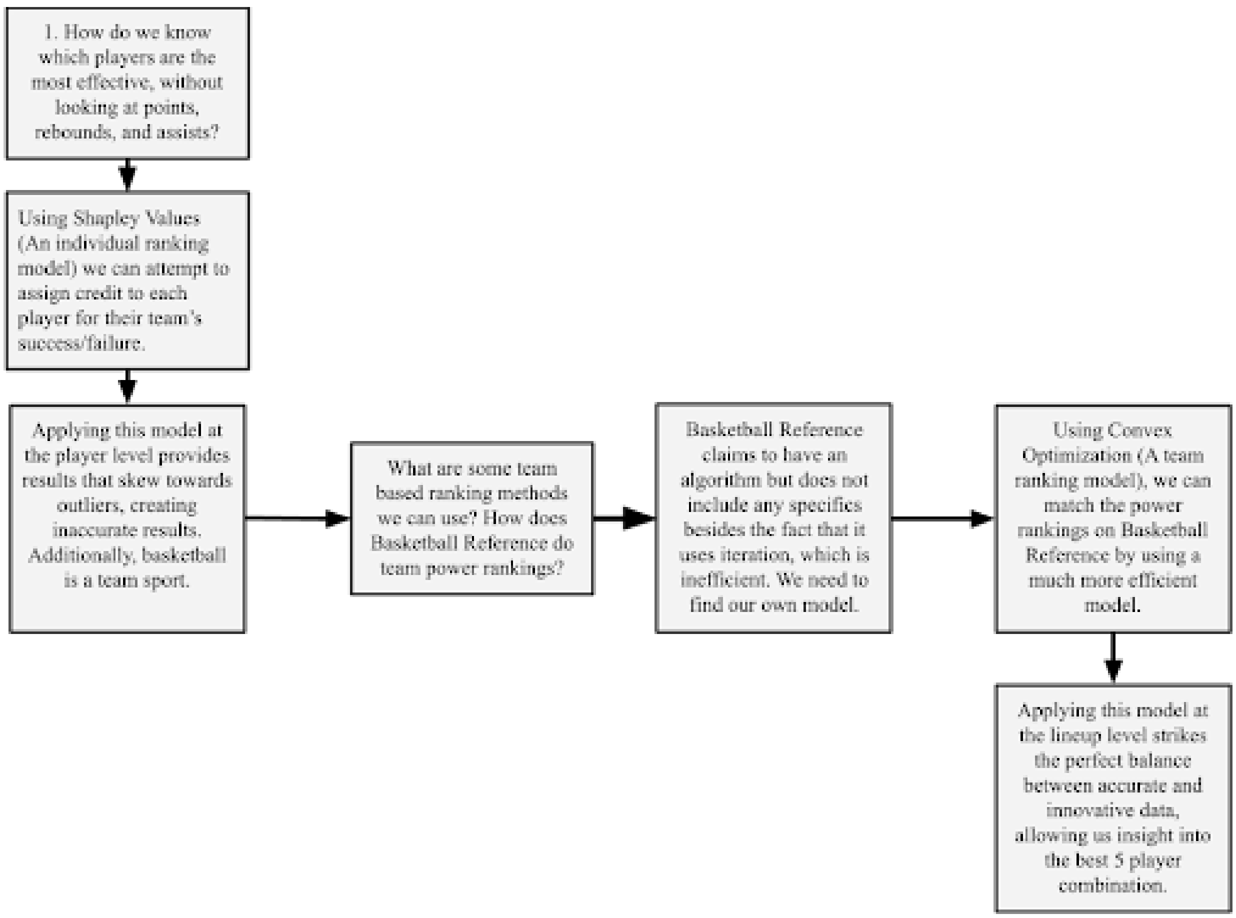

Figure 1: the stages of this study and the strengths/weaknesses of each model. Starting with Shapley Values and ending with Convex Optimization

A natural starting point is to evaluate an individual player's performance in isolation. While this is theoretically possible with the Shapley values, assuming that all possible lineups and all possible opponents can be observed, in practice, player contributions are often inferred from small samples. These calculations can be approximated by assuming that contributions are linear and additive. This motivates regressing team outcomes on a matrix of dummy variables representing specific players in a specific lineup. However, individual production does not provide any meaningful insight into team construction. A team of the five most individually dominant players is unlikely to be the most effective lineup. Instead, team-based rankings offer a more accurate framework.

When consulting reliable sources like Basketball Reference for their NBA team rankings, the available link redirects to a defunct page on Football Reference that vaguely mentions an iterative algorithm.[2] However, an explicit algorithm is not given, and even if accurate, the models discussed could potentially exhaust computational resources. A Cornell blog on the same subject also discusses misleading results produced by models exclusively based on win-loss records.[7]

A further challenge is that many existing models are inaccessible. Their exact algorithms are often described as "the gory mathematical details",[3] and are withheld hidden from a general audience, as was the case with the Basketball Reference power rankings.[2]

By utilizing convex optimization, it is possible not only to replicate the NBA team data reported on Basketball Reference but also to extrapolate that approach to rank the production of every lineup in the NBA.

3.1.1 Shapley Value

The Shapley Value provides a robust model for quantifying individual contributions to a collective outcome. It is agnostic to the functional form of each individual's influence on the team performance, allowing for nonlinear contributions and interactions between players' performances. The approach relies on measuring the marginal impact of adding a player to a variety of different lineups. If team performance improves when a particular player is included in the lineup, that player is considered to have made a contribution, regardless of if that contribution maps to conventional statistics such as points, rebounds, or blocks. The application of Shapley Values to the NBA was inspired by Metulini's and Gnecco's papers on the same subject.[5]

The formula for the Shapley value is as follows:

$$\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \tfrac{|s|!(n-|s|-1)!}{n!} (v(S \cup \{i\} - v(S))$$

$$= \tfrac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \tbinom{n-1}{|s|}^{-1} v(S \cup \{i\}) - v(S)$$

In practical application, the notation can be interpreted using relevant examples. Let *"S"* denote the size of a lineup, which is five in a basketball setting, and let *"v(S)"* represent the total points scored during possessions involving that lineup. The marginal contribution of each player is found by subtracting the production of a lineup including player i : $v(S \cup \{i\})$from the production when they are excluded, simply $v(s)$. The reciprocal factorial term (n-1 choose S) represents the number of ways a player can be added within a lineup by determining the number of ways five players can be chosen from an eleven-man roster. This term functions as a weight, allowing the calculation of a weighted average across all marginal contributions, accounting for the probability that a certain player will join a lineup. Division by n ensures each player in a lineup contributes proportionally towards the observed result. For instance, if a lineup scores 10 points, each player is credited with 2 points.

3.1.2 A Simple Model with Shapley Values

To provide a more concrete example, consider a simple model of team success, where individual player contributions are fixed and team outcomes are a linear function of the players on the floor. Specifically, *v(S)= 1a + 3b + 4c + 7d + 11e*, where *e a, b, c, d,* and e are indicator variables (1 if the player is on the court, and 0 otherwise).

To reduce the number of permutations, this hypothetical example assumes that a lineup is formed with three players, resulting in ten unique three-player lineups from a five-player roster

A matrix can be constructed to represent the results of each lineup's time on the court, using binary notation to represent which players are in each lineup. *v(S)* represents the total contribution of each lineup.

A	B	C	D	E	$v(S)$	$C = v(S)/n$
1	1	1	0	0	8	2.67
1	1	0	1	0	11	3.67
1	1	0	0	1	15	5.00
1	0	1	1	0	12	4.00
1	0	1	0	1	16	5.33
1	0	0	1	1	19	6.33
0	1	1	1	0	14	4.67
0	1	1	0	1	18	6.00
0	1	0	1	1	21	7.00
0	0	1	1	1	22	7.33

Table 1: Left: Lineups Matrix / Middle: Target Column / Right: Individual Player's Contribution

In reality, data for all possible permutations will not be accessible. In addition, the functional form of

(S) will be unknown. Only the observed output of *(S)* will be available.

Player contributions can first be estimated using the Shapley Value directly. Ideally, the marginal contribution of each player would be measurable. Since a two-player roster is not permitted, therefore *v(S/i)* is assumed to be 0, and the marginal contribution *v(S)-v(S/i)* = *v(S)*. (For simplicity, *v(S)* is defined as the lineup including player i and *v(S/i)* as the lineup excluding that player.) Furthermore, each player makes the same argument that *v(S/i)* = 0. In other words, if not for that player's presence, the team's output would be 0. Therefore, *v(S)* represents the marginal contribution, and each player's contribution is computed as C = *v(S) /n* (Table 1 right column).

A weighted average is taken across all lineups that feature that player based on the probability that that player is featured in any given lineup. In this example, each player appears in six such permutations. This corresponds to the formula above $\binom{n-1}{|s|}^{-1}$. In this case, 4 choose 2. In other words, choose two players from the remaining four players. This makes the weight for each lineup $\frac{1}{6}$. In both this scenario and in a basketball setting, N and S are constant; therefore, $\binom{n-1}{|s|}^{-1}$ is denoted as a coefficient.

Player	Contribution
A	4.50
B	4.83
C	5.00
D	5.50
E	6.17
Total	26.00

Although player contributions have the correct rank ordering, the spread between the contribution of player E and player A is much smaller than the pre-specified contribution values (1, 3, 5, 7, 11). This occurs because Shapley assigns significant value to teamwork, particularly in settings like basketball, where no one player can play alone. Additionally, due to the strict nature of the lineup size, marginal contributions cannot be perfectly determined.

An alternative method treats the matrix in the blue box as a system of equations and uses linear algebra to extract each player's contribution.

L = Lineups Matrix (Table 1 Left Column)

$$S{=}L^T L$$

Since the Lineups matrix is made up of 1s and 0s, taking the Gram matrix will create the Shared Lineups Matrix, which quantifies how frequently players appear together in different lineup combinations.

T = Target Column (Table 1 Center Column)

$$P = L^T T$$

This operation ensures that only players who were in the lineup receive credit. They will receive no credit if they are not in the lineup (i.e., there is a 0 in that column).

$$X = S^{-1} P$$

This final step reduces each player's impact on their contribution per possession, as players who played in more possessions tend to have a higher impact. This step is not important in this case since every player played an equal number of possessions.

This is the matrix that is produced:

A	1
B	3
C	4
D	7
E	11

This exact recovery is a consequence of the linearity assumption in the model, demonstrating that linear algebra models can accurately extract player contributions from linear functions like *v(S)* in this example.

3.1.3 Convex Optimization

Another intuitive algorithm that can be utilized to replicate the results of Basketball Reference's proprietary ranking system is convex optimization.[1] Rather than focusing on individual success, convex optimization evaluates the success of an entire team in head-to-head matchups.

In this redefined problem S represents a two-element set winning team, losing team. Denote the point differential for each game as $v(\{\textit{Team A, Team B}\})$. The next step is evaluating the following marginal contribution: $v(\{\textit{Team A, Team B}\})$ - $v(\{\textit{ Team B}\})$. However, $v(\{\textit{ Team B}\})$ is undefined.

There is no game with only one team, although $v(\{\textit{ Team B}\})$ is reasonably defined to mean the outcome when *Team B* plays against a typical or average opponent. $v(\{\textit{Team A, Team B}\})$ - $v(\{\textit{ Team B}\})$ is in turn the excess point differential when *Team A* plays *Team B*. For example, if *Team B* typically loses to their opponents by 5 points, and *Team A* beats *Team B* by 8 points, *Team A* has a 3-point marginal contribution (8 - 5).

It is jointly estimated that $v(\{$ *Team A*$\})$ is 3 and $v(\{$ *Team B*$\})$ is -5, from the single observation that A outscored B by 8 points. Furthermore, since this model deals strictly in point differentials rather than point totals, it is possible to add a constant to both *Team A* and *Team B* and retain the same results. For example, A = 4 = (3 + 1) and B = -4 = (-5 + 1) also model the data perfectly. Therefore, a constraint is added such that the sum of all team values = 0.

$$\sum_{i=1}^{T} v(i) = 0$$

Using convex optimization minimizes the squared error between the predicted point differential and actual point differential. The actual point differentials are directly observed from game outcomes and predicted outcomes are given by the following formula.

$$pd(i,j) = v(i) - v(j)$$

One criticism of the traditional optimization strategy is the potential failure to converge on a solution. However, a solution is guaranteed using convex optimization. Boyd and Vandenberghe's course on the subject[1] provides a framework to understand this: the optimization problem can be approached by imagining my data as a landscape, where the goal is to locate the lowest point. The mathematical formula of convex optimization guarantees that such a solution exists and can be efficiently computed. Here's a breakdown of how this optimization can be this study.

A key advantage of convex optimization in this context is that matrices provide a structured way to encode large datasets as linear functions, enabling efficient optimization. It also allows constraints to be added to the matrix itself. And by taking a Gram Matrix, a positive semi-definite matrix is obtained, which guarantees a maximum or minimum. A matrix "M" is positive semi-definite if, for any nonzero vector x, the following is true:

$$x^T M \geq 0$$

To visualize a team's record, a table is constructed with all the teams at the top. In each row, a 1 will be assigned to every winning team, a -1 will be assigned to every losing team, and 0 will be assigned to every team that did not play. Then there will be a column with the point differential for every game. Here's a toy model with 5 teams using the convex optimization framework outlined earlier. In this hypothetical, A scores 1 point every game, B scores 2, and so on.

A	B	C	D	E	Point Differential (P)
-1	1	0	0	0	1
0	-1	0	1	0	2
0	0	-1	1	0	1
0	-1	0	0	1	3
-1	0	0	0	1	4

Take the Gram Matrix of the Results Matrix (Table 3, Right Column):

R = *Results Matrix*

C = *Game Count Matrix*

$$C{=}R^T R$$

The combination of -1s, 0s, and 1s that make up the results matrix ensures that when the Gram matrix is taken, the resulting matrix will represent the number of times two teams have faced each other. The next step outlined by the convex optimization formula is to add constraints to the game count matrix, guaranteeing that it is invertible. Adding a constraint is equivalent to adding a "ground level" to base the rest of the rankings.

	A	B	C	D	E
1	2	-1	0	0	1
1	-1	3	0	-1	-1
1	0	0	1	-1	0
1	0	-1	-1	2	0
1	-1	-1	0	0	2
0	1	1	1	1	1

Find the total point difference for every team using this formula, where P is the point-differential column (Table 3, Right Column). A *0* must be added to the bottom of P to serve as a constraint:

$$D{=}R^T P$$

This aggregates all game results to produce a cumulative point differential for each team. Finally, the team rankings can be determined using:

$$X{=}C^{-1} D$$

X is the matrix of the power rankings. The rankings are normalized based on the strength of opponents, ensuring a fairer evaluation of team performance. As a final verification step, the equation is rearranged to:

$$XR = D$$

This backs up the formula as it shows that teams with higher net differentials will receive proportionally higher rankings. The results of the calculations are shown below:

CON	0
A	-2
B	-1
C	0
D	1
E	2

The results match the conditions of the hypothetical, where A would score one point, B would score two, etc. Although the points scored per team are shifted down, their values relative to each other remain the same. This is because the constraint was set to 0, meaning 0 will be the midpoint of the results.

4 Setup and Results

4.1 Trial with PHO vs OKC game: Shapley Values

Utilizing play-by-play data from Basketball Reference, a “Lineups Matrix” (Fig. 2) is constructed that details the players who were on court during each possession. From this matrix, “Shared Lineup Matrix” (Fig. 3) was constructed, which quantifies the overlapping possession between each player, allowing an estimation for player contributions per 100 possessions (Table 2,3). However, due to Shapley’s strict requirement of exhaustive lineup data, accuracy decreases when certain lineups appear infrequently or not at all. Thus, weighting adjustments were added to reduce the influence of lineups with fewer than 10 possessions.

Chet Holmgren	Kenrich Williams	Jalen Williams	Luguentz Dort	Gordon Hayward	Isaiah Joe	Bismack Biyombo	Shai Gilgeous-Alexander	Josh Giddey	Cason Wallace	Aaron Wiggins	team_id	Score	possess
1	0	1	1	1	0	0	1	0	0	0	T0101100100	9	52
1	0	1	1	1	0	0	1	0	0	0	T0101110000	5	5
0	1	0	1	1	1	0	1	0	0	0	T0011101100	7	12
0	1	0	1	1	1	0	1	0	0	0	T0011101100	1	6
0	1	1	1	0	0	0	1	0	0	0	T0011010100	3	1
0	1	1	1	0	0	0	1	0	0	0	T0011010100	2	1
0	0	0	1	0	1	1	1	1	0	0	T0001011110	19	17
1	0	0	1	0	1	0	1	1	0	0	T0101011100	2	12
1	0	0	1	0	1	0	1	1	0	0	T0101011100	2	2
0	1	0	1	1	0	0	1	0	1	0	T0011101010	4	4
1	0	0	1	0	0	0	1	0	0	1	T0101001001	4	4
1	0	1	0	0	1	0	1	0	0	0	T0100111100	2	4
1	0	1	0	0	1	0	1	0	0	0	T0100111100	2	4
1	0	1	0	0	1	0	1	0	0	0	T0100111100	4	8
1	0	1	0	0	1	0	1	0	0	0	T0100111100	2	2
0	1	1	1	0	0	0	1	1	0	0	T0011011100	2	5
0	1	1	1	0	0	0	1	1	0	0	T0011011100	8	8
0	1	1	1	0	0	0	1	1	0	0	T0011011100	2	2
0	0	1	1	0	0	0	1	0	0	1	T0001101001	5	9

Figure 2: Lineups Matrix

	Chet Holmgren	Kenrich Williams	Jalen Williams	Luguentz Dort	Gordon Hayward	Isaiah Joe	Bismack Biyombo	Shai Gilgeous-Alexander	Josh Giddey	Cason Wallace	Aaron Wiggins
Chet Holmgren	10	0	8	6	4	3	0	8	2	4	4
Kenrich Williams	5	2	2	2	3	3	0	5	1	3	1
Jalen Williams	8	2	11	5	3	3	1	9	3	5	5
Luguentz Dort	6	5	9	11	4	5	3	11	4	3	3
Gordon Hayward	4	3	3	4	7	4	0	6	1	2	1
Isaiah Joe	3	3	3	4	4	8	1	6	2	3	1
Bismack Biyombo	0	0	1	3	0	1	0	3	0	2	2
Shai Gilgeous-Alex.	8	5	9	11	6	6	3	16	4	5	5
Josh Giddey	2	1	3	4	1	2	0	4	5	2	2
Cason Wallace	4	3	5	3	2	3	2	5	2	4	4
Aaron Wiggins	4	1	5	3	1	1	2	5	2	4	7
Constrain	1	1	0	0	0	0	1	0	0	0	0

Figure 3: Shared Lineups Matrix

Key Takeaways:

Player	Poss	Point Contribution
Chet Holmgren	78	0.63
Kenrich Williams	17	5.51
Jalen Williams	78	14.70
Luguentz Dort	78	11.03
Gordon Hayward	38	5.29
Isaiah Joe	40	4.43
Bismack Biyombo	22	6.00
Shai Gilgeous-Alex.	92	31.84
Josh Giddey	45	4.91
Cason Wallace	41	15.77
Aaron Wiggins	56	11.43
		111.55

Table 2: Player Possessions vs Point Contributions (Color-Coded)

The Shapley Value struggles to isolate the individual contributions of NBA players because it requires complete data on all lineup permutations. Additionally, its assumption of equal contribution undervalues star players while overvaluing weaker players. As a result, players who often appear alongside high-impact players may receive inflated credit.

Shai Gilgeous-Alexander having the greatest impact on his team is logically sound, with Jalen Williams and Luguentz Dort making decent contributions as well. What's shocking is Chet Holmgren's production: contributing about 0 points despite having more points, rebounds, and assists, and shooting more efficiently than Dort in that game, with both playing the same amount of possessions. It wasn't a matter of defense, as this data is purely the Thunder's offensive production.

Player	Poss	Point Contribution
Jusuf Nurkic	80	-6.38
Drew Eubanks	24	9.21
David Roddy	18	5.79
Bradley Beal	82	42.52
Kevin Durant	87	24.54
Royce O'Neale	80	38.10
Grayson Allen	88	-11.05
Saben Lee	32	3.64
Eric Gordon	75	6.43
Bol Bol	19	6.50

Table 3: Player Possessions vs Point Contributions (Color-Coded)

This Defensive data is highly skewed, with heavy outliers on both ends. On a team like the Suns, no one is a standout defender. Given that both of them play roughly the same level of defense, it is unlikely that Bradley Beal was solely responsible for conceding 42 points while Grayson Allen contributed to saving 11 points. This likely results from certain plays being overemphasized, resulting in misleading conclusions about individual defensive impact.

While this model has significant potential for evaluating player contributions without relying on traditional statistics, its reliance on complete data is a heavy determinant. The requirement that every possible lineup must be present in the data makes obtaining perfect Shapley values nearly impossible, especially as group size increases.

In addition to its drawbacks, having each player's individual production is not helpful. Basketball is played with a team, so no matter how individually dominant a player is, he must share the floor with 4 other people. Assuming that the five most individually productive players on each team would make the best lineup is a fallacy, as chemistry and ball-dominance are important factors overlooked by this model. Focusing on evaluating lineups as a unit, rather than an individual player would be far more revealing.

To retain the possibility of non-linear contributions and interactions between player contributions, contributions will be measured at the lineup level rather than the player level. To address the issue of marginal contribution, convex optimization will be used to solve for the unobserved level of lineup skill. In addition, the next section introduces the concept of opponents. At this point, team success has simply been assumed to be a function of a single team, rather than a single team playing against an opponent

As a side note, a more direct approach to measuring marginal impact is A/B testing. This maintains the system of equation approach but simplifies the interpretation and implementation. Swapping two players while maintaining the same surround lineup isolates the player's contribution by taking the difference in production. This is an enhancement to traditional on-off metrics that evaluate team performance when a certain player is on the court versus when that player is off the court, but fail to control for the other players on the court.

4.2 Demonstration using 2024-2025 NBA Teams: Linear Algebra Method

This method constructs team power rankings from game outcome data. First, game results are stored in a "Game Results" matrix (wins and losses are marked with +1 and -1, respectively) (Table 4). There is a corresponding "Points Column" that denotes the point differential for each game. A Gram Matrix of this data yields the "Game Count Matrix" (Table 5), which adjusts for strength of schedule once inverted. Multiplying by the cumulative point differential ("Differential Column") produces normalized power rankings for each team. (Table 7)

diff	CLE	BOS	NYK	MIL	IND	MIA	DET	ORL	ATL	CHI	PHI	TOR	BRO	CHA	WAS	OKL	HOU	MEM	DEN	LAL	LAC	MIN	PHO	DAL	SAC	GOL	SAN	POR	NOP	UTA
23	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0
20	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Table 4: Matrix of team matchups

1	48	-2	-1	-3	-2	-2	-2	-1	-3	-2	-3	-3	-2	-3	-3	-2	-2	0	-2	-2	0	-1	-1	-1	0	-2	0	0	-2	-1
1	-2	49	-1	-3	-3	-1	-3	-2	-3	-4	-1	-3	-2	-2	-3	-1	-2	-1	-1	-1	-2	-2	0	-1	-1	-2	0	0	-2	-1
1	-1	-1	48	-2	-2	-1	-3	-4	-3	-2	-2	-3	-3	-2	-3	-2	-1	-1	-2	0	0	-2	-1	-1	-1	0	-1	0	-2	-2
1	-3	-3	-2	46	-2	-2	-2	-3	-2	-4	-2	-3	-4	-2	-2	0	-1	-1	0	0	-1	0	0	0	-1	0	-2	-2	0	-2
1	-2	-3	-2	-2	46	-3	-4	-3	0	-2	-3	-2	-2	-2	-1	-1	-1	-1	0	0	0	0	-2	-1	-1	-2	-2	-1	-3	0
1	-2	-1	-1	-2	-3	46	-3	-4	-1	0	-1	-3	-2	-2	-1	-1	-1	0	-2	-2	-1	-1	-2	-1	-2	-1	-1	-2	-1	-2
1	-2	-3	-3	-2	-4	-3	48	-3	-2	-1	-2	-3	-2	-3	-1	0	-2	-1	-1	-2	0	-1	-2	-1	-1	-1	0	-1	0	-1
1	-1	-2	-4	-3	-3	-4	-3	49	0	-2	-4	-2	-4	-2	-1	-2	0	-1	-1	-1	-1	-1	-2	-1	0	0	0	-2	-1	-1
1	-3	-3	-3	-2	0	-1	-2	0	48	-4	0	-3	-1	-2	-3	-1	-1	-1	-2	-2	-1	-2	-2	-1	-2	-1	-1	-1	-2	-1
1	-2	-4	-2	-4	-2	0	-1	-2	-4	49	-2	-2	-2	-3	-3	-1	-1	-2	-1	0	-1	-1	0	-1	-1	-1	-2	-1	-2	-1
1	-3	-1	-2	-2	-3	-1	-2	-4	0	-2	47	-1	-2	-4	-1	-1	-1	-2	-2	-2	-2	0	-2	0	-2	-1	-1	-1	-1	-1
1	-3	-3	-3	-3	-2	-3	-3	-2	-3	-2	-1	48	-2	-1	-1	-1	-1	-1	-2	-2	-1	-2	0	-1	-2	-1	0	0	-2	0
1	-2	-2	-3	-4	-2	-2	-2	-4	-1	-2	-2	-2	48	-2	0	-1	0	-3	-2	-1	-1	0	-2	0	-2	-1	-1	-1	-1	-2
1	-3	-2	-2	-2	-2	-2	-3	-2	-2	-3	-4	-1	-2	45	-2	-1	-2	-1	0	-1	-1	-1	-2	-1	0	0	0	-1	-1	-1
1	-3	-3	-3	-2	-1	-1	-1	-1	-3	-3	-1	-1	0	-2	47	-2	-2	-2	-1	-2	-2	-1	-2	-2	-1	-2	-1	0	-2	0
1	-2	-1	-2	0	-1	-1	0	-2	-1	-1	-1	-1	-1	-1	-2	46	-3	-1	-2	-1	-3	-1	-1	-4	-1	-3	-2	-3	-2	-2
1	-2	-2	-1	-1	-1	-1	-2	0	-1	-1	-1	-1	0	-2	-2	-3	47	-4	-1	-1	-3	-2	0	-2	-2	-3	-3	-3	-2	0
1	0	-1	-1	-1	-1	0	-1	-1	-1	-2	-2	-1	-3	-1	-2	-1	-4	48	-2	-3	-1	-2	-1	-2	-2	-3	-2	-2	-3	-2
1	-2	-1	-2	0	0	-2	-1	-1	-2	-1	-2	-2	-2	0	-1	-2	-1	-2	48	-1	-4	-2	-2	-4	-2	-1	-2	-1	-2	-3
1	-2	-1	0	0	0	-2	-2	-1	-2	0	-2	-2	-1	-1	-2	-1	-1	-3	-1	46	-1	-3	-3	-1	-4	-2	-3	-2	-1	-2
1	0	-2	0	-1	0	-1	0	-1	-1	-1	-2	-1	-1	-1	-2	-3	-3	-1	-4	-1	48	-3	-3	-2	-2	-3	-3	-3	-1	-2
1	-1	-2	-2	0	0	-1	-1	-1	-2	-1	0	-2	0	-1	-1	-1	-2	-2	-2	-3	-3	48	-2	-3	-3	-4	-3	-3	-1	-1
1	-1	0	-1	0	-2	-2	-2	-2	-2	0	-2	0	-2	-2	-2	-1	0	-1	-2	-3	-3	-2	47	-3	-2	-3	-1	-2	-1	-3
1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	0	-1	0	-1	-2	-4	-2	-2	-4	-1	-2	-3	-3	49	-1	-2	-2	-4	-3	-3
1	0	-1	-1	-1	-1	-2	-1	0	-2	-1	-2	-2	-2	0	-1	-1	-2	-2	-2	-4	-2	-3	-2	-1	47	-2	-3	-2	-1	-3
1	-2	-2	0	0	-2	-1	-1	0	-1	-1	-1	-1	-1	0	-2	-3	-3	-3	-1	-2	-3	-4	-3	-2	-2	48	-1	-1	-3	-2
1	0	0	-1	-2	-2	-1	0	0	-1	-2	-1	0	-1	0	-1	-2	-3	-2	-2	-3	-3	-3	-1	-2	-3	-1	45	-3	-1	-4
1	0	0	0	-2	-1	-2	-1	-2	-1	-1	-1	0	-1	-1	0	-3	-3	-2	-1	-2	-3	-3	-2	-4	-2	-1	-3	48	-47	-2
1	-2	-2	-2	0	-3	-1	0	-1	-2	-2	-1	-2	-1	-1	-2	-2	-2	-3	-2	-1	-1	-1	-1	-3	-1	-3	-1	-4	49	-2
1	-1	0	-2	-2	0	-2	-1	-1	-1	-1	-1	0	-2	-1	0	-2	0	-2	-3	-2	-2	-1	-3	-3	-3	-2	-4	-2	-2	46
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 5: Game Count Matrix

	CLE	BOS	NYK	MIL	IND	MIA	DET	ORL	ATL	CHI	PHI	TOR	BRO	CHA	WAS	OKL	HOU	MEM	DEN	LAL	LAC	MIN	PHO	DAL	SAC	GOL	SAN	POR	NOP	UTA	
CLE	0.3333	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0.03	0	0.03	0.03	0.03	0	0	0	0.03	0	0	0	0	0.03	0.03	0	-6.9E-16
BOS	0.01975	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
NYK	-0.0003	0	-0	0	-0	-0	-0	-0	-0	0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
MIL	-0.0008	-0	0	-0	-0	-0	0	0	-0	-0	-0	0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
IND	3.3E-05	0	-0	0	-0	-0	-0	0	-0	0	-0	0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
MIA	-0.0004	-0	-0	-0	0	0	0	0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
DET	-0.0005	-0	-0	-0	0	0	0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
ORL	-0.0004	-0	0	-0	0	0	0	0	-0	-0	-0	0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
ATL	-0.0008	-0	0	0	0	0	0	0	-0	-0	0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
CHI	-4E-05	-0	-0	-0	-0	-0	-0	-0	0	0	-0	-0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
PHI	-0.0004	0	-0	0	-0	-0	-0	-0	0	0	-0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
TOR	-7E-05	-0	-0	-0	0	-0	-0	0	-0	-0	0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
BRO	2e-05	0	0	0	-0	-0	0	-0	-0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
CHA	-0.0005	-0	0	0	-0	-0	-0	0	-0	-0	-0	-0	0.02	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
WAS	8.5E-05	-0	-0	-0	-0	-0	0	-0	-0	0	0	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
OKL	-4E-05	-0	-0	-0	-0	-0	-0	-0	0	-0	-0	-0	-0	-0	0.02	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
HOU	-0.0007	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0	-0	-0	-0	0	-0	-0	0	-0	0	-0	0	-0	-0	0.033333
MEM	-0.0006	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0.02	0	-0	-0	0	-0	-0	-0	-0	0	0	0	-0	-0	0.033333
DEN	-0.0014	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0.02	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.033333
LAL	-0.0007	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0.02	-0	0	-0	-0	0	-0	-0	-0	-0	-0	0	0.033333
LAC	-0.0007	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	0	0	-0	0	-0	0	-0	-0	-0	0.033333
MIN	-0.0015	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0	-0	0	-0	0	0	-0	-0	-0	0	0	0	-0	-0	0.033333
PHO	-0.0011	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0	0	-0	0	0	0	0	0	-0	-0	0.033333
DAL	-0.001	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	-0	0.02	-0	-0	-0	-0	-0	-0	0	0.033333
SAC	-0.0011	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	-0	0	-0	-0	0	-0	0	-0	-0	-0	0	-0	0	0.033333
GOL	-0.0015	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	0	-0	-0	0	-0	0	-0	-0	0	0.033333
SAN	-0.0007	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0	-0	-0	-0	0	0	-0	-0	-0	0	-0	-0	-0	-0	0.033333
POR	-0.0016	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	-0	0	0	0	-0	-0	0	-0	0	0	-0	0	0.033333
NOP	-0.0015	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0	-0	-0	-0	0	0	-0	0	-0	-0	0	0.02	0	-0	0.033333
UTA	-0.0006	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0.02	-0	0.033333
CON	-0.0011	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	-0	-0	-0	0	0	0	-0	0	-0	-0	0	0.033333

Table 6: Game Count Inverse Matrix

	Team Diff		Power
CLE	488	CON	0.00
BOS	433	CLE	8.92
NYK	338	BOS	7.96
MIL	115	NYK	5.53
IND	48	MIL	1.21
MIA	7	IND	0.57
DET	-58	MIA	-0.47
ORL	-70	DET	-1.20
ATL	-153	ORL	-1.79
CHI	-161	ATL	-3.17
PHI	-165	CHI	-3.63
TOR	-258	PHI	-3.37
BRO	-348	TOR	-4.41
CHA	-231	BRO	-6.71
WAS	-716	CHA	-5.30
OKL	544	WAS	-13.47
HOU	261	OKL	11.75
MEM	362	HOU	6.57
DEN	193	MEM	6.60
LAL	-22	DEN	4.36
LAC	160	LAL	-0.70
MIN	155	LAC	3.65
PHO	-20	MIN	3.89
DAL	143	PHO	-1.22
SAC	92	DAL	3.25
GOL	-15	SAC	1.61
SAN	-52	GOL	0.76
POR	-303	SAN	-0.75
NOP	-408	POR	-5.57
UTA	-359	NOP	-7.78
CON	0	UTA	-7.11

Table 7: Point Differential Column Relative strengths of each NBA team during the 2024-2025 season. .

Key Takeaways:

The convex optimization Model, which involves using Linear Algebra, resolves many issues seen in other ranking approaches. It accounts for the frequency of matchups, a factor that is overlooked in iterative optimization models. Additionally, by using a system of equations, it neutralizes the effects of outliers and blowouts, as extreme results do not skew the final result.

Mathematically, the convex optimization model is also sound. Unlike black-box optimization techniques that rely on methods that may not always converge on a solution, this approach guarantees a unique, solvable solution. Additionally, this model requires much less computing power.

One minor issue with this model is the lack of home and away adjustments; there are no distinguishing factors that separate the home and away teams. However, this can be easily resolved by considering the home and away versions of each team as two separate entities and finding the power rankings for 60 teams instead. Overall, most weaknesses of this model are minor or easily fixable.

Looking at the results of the model and comparing them to the real-life success of the NBA just verifies its validity. The Thunder and the Cavaliers are at the top of the power rankings. This is accurate because those two teams are at the top of their respective conferences. Once again, comparing the results to the power ranking data found on Basketball Reference reveals that the model was able to replicate their numbers to within 0.01. However, convex optimization is superior as it uses a set formula, while Basketball Reference uses iteration, a more computationally taxing method.

4.3 Power Rankings with Lineups

This method applies the same ideas of convex optimization used for team rankings, but using lineups instead of teams. Unique IDs are assigned to every lineup across the course of the entire season. The "Matchup Matrix" (Table 8) details possessions between opposing lineups. In this model, each possession is treated as a game. Point Differentials are aggregated to a "Total pm" column. Multiplying this by the inverse of the Matchup Matrix yields (Table 9) lineup power rankings.(Table 11)

To improve reliability, only the 200 most common lineups were retained while the rest where aggregated to 30 "bench" lineups (one per team). This prevents lineups with little data from disrupting the data while still capturing the strength of every team's rotation.

	-22639	-22636	-22630	-22627	-22626	-22618	-22617	-22616	-22612	-22605
-22639	16561	-279	-189	-329	-586	-135	-467	-682	-348	-325
-22636	-279	13160	-501	-378	-284	-456	-272	-258	-442	-468
-22630	-189	-501	14256	-468	-269	-439	-313	-260	-304	-1131
-22627	-329	-378	-468	14785	-341	-762	-191	-332	-397	-457
-22626	-586	-284	-269	-341	16747	-136	-661	-732	-399	-301
-22618	-135	-456	-439	-762	-136	12042	-251	-217	-1005	-182
-22617	-467	-272	-313	-191	-661	-251	13863	-622	-212	-320
-22616	-682	-258	-160	-332	-732	-217	-622	18393	-256	-282
-22612	-348	-442	-304	-397	-399	-1005	-212	-256	13573	-725
-22605	-325	-468	-1131	-457	-301	-182	-320	-282	-725	20915
-22604	-296	-205	-241	-288	-429	-304	-535	-743	-109	-359
-22601	-770	-331	-206	-253	-571	-284	-358	-751	-260	-436
-22600	-483	-529	-610	-672	-321	-483	-337	-444	-626	-903
-22598	-604	-319	-329	-427	-556	-105	-690	-542	-266	-213
-22597	-519	-355	-284	-137	-652	-112	-354	-633	-54	-687
-22596	-468	-359	-215	-285	-570	-269	-604	-567	-252	-453

Table 8: Matchup Matrix

	-22639	-22636	-22630	-22627	-22626	-22618	-22617	-22616	-22612	-22605	-22604	-22601	-22600	-22598	-22597	-22596	-22592	-22587
-22639	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22636	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22630	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22627	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22626	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22618	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22617	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22616	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22612	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22605	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22604	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22601	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22600	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
-22598	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
-22597	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
-22596	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
-22592	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
-22587	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
-22571	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22569	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22568	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22567	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22552	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22544	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22503	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22480	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22478	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22295	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-22293	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9: Matchup Matrix Inverse

Power
-0.05
-0.01
-0.01
-0.01
-0.06
-0.01
-0.02
-0.01
-0.01
-0.01
-0.01
-0.03
-0.04
-0.01
0.02
-0.05
-0.02
0.01
-0.03

Table 10: Lineup Power Rankings

Key Takeaways

This proves that convex optimization can be applied on multiple different levels, either at the team level or the lineup level. These results provide insight into not only which teams excel but also which lineups should be used. Many of the top-ranking lineups are not each team's starting lineup, which means that many NBA coaches do not play their best lineups the most often.

Another strength of this model is that it is extremely simple. It only requires basic on/off data and point differential stats to make this model work, meaning this could hypothetically even be used at the high school level. These formulas are not complicated and are easily accessible to most people with a simple statistics background.

Now, let's look at the actual results to see if the model's results reflect real-life data.

Joel Embliid	Tyrese Maxey	Nicolas Batum	Tobias Harris	De'Anthony Melton	PHI	12.94
Jrue Holiday	Jayson Tatum	payton Pritchard	Sam Hauser	Al Horford	BOS	11.16
Zion Williamson	Larry Nance Jr.	Naji Marshall	Jose Alvarado	Trey Murphy III	NOP	11.15
Luguentz Dort	Jalen Williams	Chet Holmgren	Shai Gilgeous-Alexander	Isaiah Joe	OKC	10.02
Andrew Wiggins	Klay Thompson	Trayce Jackson-Davis	Draymond Green	Stephen Curry	GSW	9.98
Joel Embiid	Kyle Lowry	Kelly Oubre Jr.	Tyrese Maxey	Tobias Harris	PHI	9.97
Julius Randle	Jalen Brunson	Immanuel Quickley	Isaiah Hartenstein	Josh Hart	NYK	9.75
Dereck Lively II	Luka Doncic	Kyrie Irving	P.J. Washington	Josh Green	DAL	9.41
Rudy Gobert	Naz Reid	Nickeil Alexander-Walker	Anthony Edwards	Kyle Anderson	MIN	8.76
Giannis Antetokounmpo	Damiam Lillard	Pat Connaughton	Malik Beasley	Brook Lopez	MIL	8.76
Kevon Looney	Andrew Wiggins	Klay Thompson	Chris Paul	Stephen Curry	GSW	8.72
Jalen Brunson	Donte DiVincenzo	Isaiah Hartenstein	Josh Hart	Miles McBride	NYK	8.61
Zion Williamson	Jonas Valanciunas	Brandon Ingram	Dyson Daniels	Herbert Jones	NOP	8.57
Jrue Holiday	Jayson Tatum	Payton Pritchard	Sam Hauser	Luke Kornet	BOS	8.09
Dereck Lively II	Luka Doncic	Kyrie Irving	Derrick Jones Jr.	Josh Green	DAL	7.95
Victor Wembanyama	Jeremy Sochan	Keldon Johnson	Devin Vassell	Tre Jones	SAS	7.90
Kawhi Leonard	Mason Plumlee	Paul George	James Harden	Terance Mann	LAC	7.65
Kristaps Porzingis	Jayson Tatum	Derrick White	Al Horford	Jaylen Brown	BOS	7.55
Kawahi Leonard	Mason Plumlee	Paul George	James Harden	Terance Mann	LAC	7.49
DeMar DeRozan	Nikola Vucevic	Ayo Dosunmu	Coby White	Patrick Williams	CHI	7.48
Naz Reid	Nickeil Alexander-Walker	Anthony Edwards	Karl-Anthony Towns	Jaden McDaniels	MIN	7.41
CJ McCollum	Brandon Ingram	Larry Nance Jr.	Herbert Jones	Trey Murphy III	NOP	7.23
Isaac Okoro	Max Strus	Donovan Mitchell	Dean Wade	Jarrett Allen	CLE	7.10
Dereck Lively II	Luka Doncicc	Kyrie Irving	Derrick Jones Jr.	P.J. Washington	DAL	7.08
Damian Lillard	Pat Connaughton	Bobby Portis	Malik Beasley	Brook Lopez	MIL	7.07
Josh Giddey	Cason Wallace	Jalen Williams	Chet Holmgren	Shai Gilgeous-Alexander	OKC	6.77
Bogdan Bogdanovic	Clint Capela	Dejounte Murray	Saddiq Bey	De'Andre Hunter	ATL	6.46
Kristaps Porzingis	Derrick White	Sam Hauser	Al Horford	Jaylen Brown	BOS	6.38
Moritz Wagner	Cole Anthony	Franz Wagner	Gary Harris	Jonathan Isaac	ORL	6.05

Table 11: Top 30 Lineups

This is a list of the top 30 lineups.

One notable result is that the Philadelphia 76ers possess the highest-ranked lineup. While this might come as a surprise, it is often overlooked that they have two all-star-level players in Joel Embiid and Tyrese Maxey. Embiid, a former MVP, and Maxey, an elite scorer, form an exceptionally dominant duo when on the floor together. The factor that hampers the 76ers' success is injuries, with Joel Embiid only playing 39 games last season. Had he been available for a full campaign, the 76ers' overall performance could have been significantly stronger, perhaps being the number one team, as the results suggest. The box plus-minus score found on the[9] also supports this claim, with this lineup being +33.2 in point differential.

Rank	Lineup	Team	Minutes Played	Field Goal	Field Goal Attempted	FG%	3 Pointers	3 Pointers Attempted	3P %	effective FG%	Free Throw	Free Throw Attempted	FT %	Points
1	N. Batum / J. Embiid / T. Harris / T. Maxey / D. Melton	PHI	219:15	+8.8	-1.6	+0.105	+6.3	+8.1	+0.102	+0.141	+9.4	+10.0	+0.023	+33.2
2	N. Batum / J. Embiid / T. Harris / T. Maxey / K. Oubre	PHI	135:45	+1.3	-4.2	+0.037	-0.1	-7.2	+0.079	+0.040	+3.4	+1.5	+0.081	+5.9
3	J. Embiid / T. Harris / T. Maxey / D. Melton / K. Oubre	PHI	109:01	-5.2	-7.5	-0.017	-4.8	-2.5	-0.121	-0.039	+14.6	+16.2	+0.036	-0.7
4	R. Covington / J. Embiid / T. Harris / T. Maxey / D. Melton	PHI	67:57	-6.1	-7.9	-0.024	-3.2	-11.0	+0.033	-0.035	+3.2	-2.7	+0.232	-12.3
5	J. Embiid / T. Harris / T. Maxey / D. Melton / P. Tucker	PHI	49:25	+2.6	-14.8	+0.108	+8.8	+8.7	+0.191	+0.171	+9.8	+8.7	+0.140	+23.8
6	N. Batum / J. Embiid / K. Lowry / T. Maxey / K. Oubre	PHI	30:00	+2.7	-22.6	+0.137	+6.7	-5.9	+0.220	+0.190	-0.2	+4.9	-0.222	+11.8
7	P. Beverley / J. Embiid / T. Harris / T. Maxey / K. Oubre	PHI	25:56	+16.0	+18.0	+0.085	+1.6	-5.0	+0.118	+0.085	-2.7	-0.9	-0.083	+30.9
8	P. Beverley / J. Embiid / T. Maxey / M. Morris / K. Oubre	PHI	25:01	+16.9	+0.5	+0.192	+5.2	-1.3	+0.158	+0.221	+4.6	+6.0	+0.020	+43.7
9	P. Beverley / J. Embiid / T. Harris / D. House / K. Oubre	PHI	21:34	-15.1	+8.7	-0.199	-12.6	-11.4	-0.257	-0.271	+5.6	+5.9	+0.050	-37.3
10	P. Beverley / J. Embiid / T. Harris / T. Maxey / D. Melton	PHI	21:30	+0.7	+11.9	-0.051	+0.5	+22.2	-0.157	-0.053	-13.0	-13.0	0.000	-11.0

Table 12: Basketball Reference's Box Plus Minus

However, having the best lineup may not guarantee status as the number one team. Teams such as the Oklahoma City Thunder and the Boston Celtics, who were number one in their respective conferences, have multiple lineups in the top 30. This suggests that winning comes from having deep rosters, rather than individual star power. This suggests that individual stats and solving for production in isolation is not the most effective way to win basketball games. While both of these teams have star players, they both have stellar supporting casts contributing to greater team success. Boston's "bench" team ranks 73rd, and Oklahoma's ranks 85th. These are the two highest-ranked benches and even out-rank some teams' second or third-most-used lineups.

This principle also explains why lineups from mediocre or struggling teams sometimes rank unexpectedly high. Many of these teams lack depth, and their bench production is often a significant weakness. Additionally, some of these top lineups often feature stars who are frequently injured, such as the 76ers lineup from before and the third-ranked Pelicans lineup. The Pelicans lineup features Zion Williamson, a former first-overall pick and All-Star. However, he has been struggling with injuries in recent years. Despite that, this model suggests that when he is on the court, he contributes to a winning team.

While this model is accurate, there might be limitations in its approach. In many cases, NBA coaches are already aware of their most effective lineups, but the lack of depth remains an issue out of their control. Additionally, injuries play a key role in determining which lineups can be played. However, there are a few teams, like the Minnesota Timberwolves, that would benefit from the insights of this model. Their top-ranked lineup is not their starting lineup. This model suggests that their sixth man of the year winner, Naz Reid, should be starting, which is also demonstrated by his on-court success even when he comes off the bench. By increasing the usage of this particular lineup, the Timberwolves may have improved their overall record, demonstrating how data-driven lineup optimization can directly influence game outcomes.

This leads to the Multi-armed bandit problem. An effective coach will exploit their best line-ups while simultaneously exploring alternative line-ups that might have better performance. This exploration could be accomplished with A/B testing. Start with a set of core players and substitute one by one. Rotate role-players and back-up players to assess incremental contribution.

5 Applications

Beyond basketball, these quantitative fairness principles can be applied to any competitive team setting. This includes decomposing portfolio performances into contributions from individual investment decisions. Another field in which this is relevant is the AI market, in which the productivity of different algorithms can be tested and ranked using these models.[8] This provides insight into which algorithms should be pursued.

6 Conclusion

One of a team's most effective ways of winning is to maximize playing time for its most impactful players. Yet, traditional statistics like points, rebounds, and assists often overlook impactful players like Shane Battier, who do not generate traditional statistics.

While Shapley Values offer a theoretical framework for ranking contributions, they face practical limitations; they require complete data for all possible lineups and misassigns credit by assuming basketball is a purely additive game. They also ignore team dynamics and chemistry.

Team based ranking systems exist, but the models that sources like Basketball Reference rely on are vague and computationally taxing. In contrast, convex optimization provides a formulaic, linear algebra-based method that matches basketball reference rankings while adjusting for strength of schedule. It can also be applied at the lineup level. It is rigorous, yet accessible enough that even high school coaches can apply it using basic plus-minus statistics. One weakness of convex optimization is that it requires head-to-head data, meaning contributions can only be derived through competition. Future work can incorporate multi-armed bandit algorithms and regression based A/B testing to balance testing new lineups while playing productive ones.

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